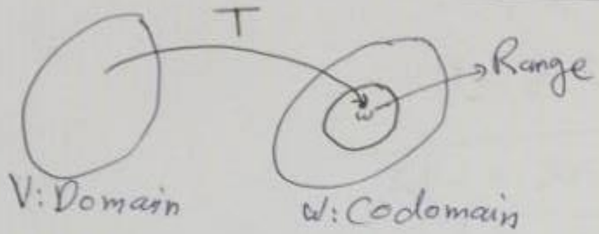
math 251 - ch 8 - week 12 Linear Transformation



$T: V \longrightarrow W$

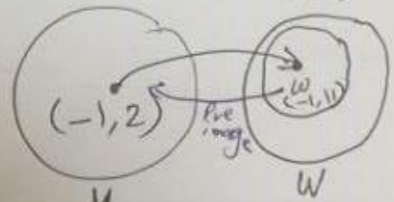
Ex1 $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, $v = (v_1, v_2) \in \mathbb{R}^2$

$T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2) \quad \cdots\cdots (1)$

(a) find the image of $v = (-1, 2)$.

(b) find the Pre-image of $w = (-1, 11)$.

Sol. (a) $v = (-1, 2)$



$T(v) = T(-1, 2)$

$= (-1 - 2, -1 + 2 \times 2)$ from (1)

$T(v) = (-3, 3)$

(b) $T(v) = w = (-1, 11)$

we know that $T(v_1, v_2) = (v_1 - v_2, v_1 + 2v_2)$

$T(v_1, v_2) = (-1, 11)$

So, $v_1 - v_2 = -1$

$-v_1 \pm 2v_2 = \pm 11$

$-3v_2 = -12$

$\boxed{v_2 = 4}$

$v_1 = -1 + v_2 = -1 + 4 \Rightarrow \boxed{v_1 = 3}$

$(3,4)$ is the Pre-image of $w = (-1, 11)$

Linear transformation:

① If $T(u + v) = T(u) + T(v)$ ← addtive Property

② $T(cu) = c\,T(u)$ (Multiplication) Property

Ex) Verify a linear transformation

T from R^2 to R^2

$T(v_1, v_2) = (v_1 - v_2,\ v_1 + 2v_2)$ ---- ①

Sol. let $u = (u_1, u_2)$ & $v = (v_1, v_2)$

and c is constant

$$u + v = (u_1, u_2) + (v_1, v_2)$$

$$= (u_1 + v_1, u_2 + v_2)$$

$$T(u+v) = T(\underbrace{u_1 + v_1}_{v_1}, \underbrace{u_2 + v_2}_{v_2})$$

$$= (u_1 + v_1) - (u_2 + v_2), u_1 + v_1 + 2(u_1 + v_2)$$

$$= u_1 - u_2 + v_1 - v_2, u_1 + 2u_2 + v_1 + 2v_2$$

$$= (u_1 - u_2, u_1 + 2u_2) + (v_1 - v_2, v_1 + 2v_2)$$

$$= T(u) + T(v)$$

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② Scalar multiplication

$$cu = c(u_1, u_2) = (cu_1, cu_2)$$

$$T(cu) = T(\underbrace{cu_1}_{v_1}, \underbrace{cu_2}_{v_2})$$

$$= (cu_1 - cu_2, cu_1 + 2u_2c)$$

$$= c(u_1 - u_2, u_1 + 2u_2)$$

$$= cT(u)$$

$\underline{\underline{T}}$ is a <u>linear transformation</u>

Ex| let a $T: R^3 \to R^3$ be a L.T such that

$$T(1,0,0) = (2,-1,4)$$

$$T(0,1,0) = (1,5,-2)$$

$$T(0,0,1) = (0,3,1)$$

find $T(2,3,-2)$

Sol. we can write

$$(2,3,-2) = 2(1,0,0) + 3(0,1,0) - 2(0,0,1)$$

$$T(2,3,-2) = T2(1,0,0) + T3(0,1,0) - T2(0,0,1)$$

$$= 2\underbrace{T(1,0,0)}_{\downarrow} + 3\underbrace{T(0,1,0)}_{\downarrow} - 2\underbrace{T(0,0,1)}_{\downarrow}$$

$$= 2.(2,-1,4) + 3.(1,5,-2) - 2.(0,3,1)$$

$$= (4,-2,8) + (3,15,-6) - (0,6,2)$$

$$= (7,7,0)$$

Ex| Show that $T(A) = A^T$ ① is a L.T

Sol. ① addition ② Scalar

1 Addition:

$$T(A+B) = (A+B)^T \quad \text{.... from ①}$$

$$= A^T + B^T$$

$$= T(A) + T(B)$$

2 Scalar Multiplication

$T(kA) = (kA)^T$ from ①

$= k.A^T$

$= k\,T(A)$

Ex) Show that $T: R^3 \longrightarrow R^3$ given by

$T(x,y) = (x,0)$ —① is a linear Trans.

sol. let $v_1 = (x_1, y_1)$ $v_2 = (x_2, y_2)$

$v_1 + v_2 = (x_1, y_1 + x_2, y_2)$

$= (x_1 + x_2, y_1 + y_2)$

addition

$T(v_1 + v_2) = T(\underbrace{x_1 + x_2}_{x}, \underbrace{y_1 + y_2}_{y})$

$= (x_1 + x_2, 0)$ from ①

$$= (x_1, 0) + (x_2, 0)$$

$$= T(x_1, y_1) + T(x_2, y_2)$$

$$= T(v_1) + T(v_2)$$

$\boxed{2}$ <u>Scalar Multiplication</u>:

$$T(cv) = T(c(x, y))$$

$$= T(\underbrace{cx}_{x}, \underbrace{cy}_{y})$$

$$= (cx, 0) \quad \text{from } ①$$

$$= cT(x, y)$$

$$= cT(v)$$

T is a linear Map.

Ex) $T: R^2 \to R^2$ where

$$T(x,y) = (x^2, y) \quad \text{——} \; ①$$

is a L.T

Sol. let $v_1 = (x_1, y_1)$ $v_2 = (x_2, y_2)$

① Addition

$$T(v_1 + v_2) = T((x_1, y_1), (x_2, y_2))$$

$$= T(\underbrace{x_1 + x_2}_{x}, \underbrace{y_1 + y_2}_{y})$$

$$= ((x_1 + x_2)^2, y_1 + y_2) \quad \text{from } ①$$

$$\neq (x_1^2, y_1) + (x_2^2, y)$$

So, T does not present the additivity.

$\to$ bcz, $(x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$

$$\neq x_1^2 + x_2^2$$

Ex) find the Standard Matrix for the L.T.

basis

$T: R^3 \longrightarrow R^2$ by

$$T(x,y,z) = (x-2y,\ 2x+y) \quad (1)$$

Sol.

$$T(1,0,0) = (1-2\times 0,\ 2\times 1+0) = (1,2)$$

$$T(0,1,0) = (0-2\times 1,\ 2\times 0+1) = (-2,1)$$

$$T(0,0,1) = (0-2\times 0,\ 2\times 0+0) = (0,0)$$

In the matrix notation is:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$